11.5 ALTERNATING SERIES TEST

IF $\{a_n\}$ IS POSITIVE, DECREASING AND CONV TO 0,

THEN $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ CONV

$\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 + \cdots$

$a_1 - a_2 + a_3 - a_4 + \cdots$ OPPOSITES

eg.

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$a_n = \frac{1}{n}$ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$f(x) = \frac{1}{x} \to f'(x) = -\frac{1}{x^2} < 0$ FOR $x \in [1,\infty)$

$f(x) = \frac{1}{x}$ DECR

$a_n = \frac{1}{n}$ DECR FOR ~~$n \in [1,\infty)$~~

~~ie.~~ $n \in \mathbb{Z}^+$

CONV

(ALT SERIES TEST)

NOTE: $-\frac{1}{1} + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots$ CONV

$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ DIV

DOES $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ CONV/DIV ?

IF $n \in \mathbb{Z}^+$,

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

$$6! = 6\cdot\underbrace{5\cdot 4\cdot 3\cdot 2\cdot 1}\, = 720$$

$$6! = 6\cdot 5!$$

$$n! = n\cdot(n-1)!$$

eg. $2n! = 2\cdot(n!)$

$$(2n)! = (2n)(2n-1)\cdots 3\cdot 2\cdot 1$$

$$\boxed{a_n = \frac{1}{n!} > 0}$$

$$0 < \frac{1}{n!} = \frac{1}{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1} \leq \frac{1}{n}$$

$$\lim_{n\to\infty} 0 = \lim_{n\to\infty} \frac{1}{n} = 0$$

BY SQUEEZE TH'M FOR SEQ

$$\boxed{\lim_{n\to\infty} \frac{1}{n!} = 0}$$

$$a_n = \frac{1}{n!} \qquad a_{n+1} = \frac{1}{(n+1)!} = \frac{1}{(n+1)\cdot n!}$$

SINCE $n \geq 1$

$$\frac{1}{n+1} < 1$$

$$\frac{1}{(n+1)n!} < \frac{1}{n!}$$

$\boxed{a_{n+1} < a_n}$ FOR $n \geq 1$

SO $\left\{\frac{1}{n!}\right\}$ IS DECR

SO $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ CONV

(ALT SERIES TEST)

DOES $\sum_{n=1}^{\infty} \frac{(-1)^n 2n}{3n+1}$ CONV/DIV?

p-SERIES
GEOMETRIC
DIVERGENCE
ALT SERIES $\rightarrow$
LIMIT COMP
COMPARISON
INTEGRAL

$$\lim_{n\to\infty} \frac{2n}{3n+1} = \frac{2}{3} \not\to 0$$

SO, $\sum_{n=1}^{\infty} \frac{(-1)^n 2n}{3n+1}$ DIV (DIV TEST)

DOES $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt[3]{n}}$ CONV/DIV?

$\{\cos n\pi\} = \{-1, 1, -1, 1, -1, 1, \ldots\}$

$\cos n\pi = (-1)^n$

$$= \boxed{\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}}$$

$\boxed{\frac{1}{\sqrt[3]{n}} > 0}$, $\boxed{\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0}$,

LET $f(x) = x^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}x^{-\frac{4}{3}} < 0$ FOR $x \geq 1$

$\left\{\frac{1}{\sqrt[3]{n}}\right\}$ IS DECR FOR $n \in \mathbb{Z}^+$

$\boxed{\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt[3]{n}} \text{ CONV}}$ (ALT SERIES TEST)

OR

$$\frac{1}{\sqrt[3]{n}} > \frac{1}{\sqrt[3]{n+1}}$$

$a_n > a_{n+1}$

$\{a_n\}$ DECR

11.6 ABSOLUTE + CONDITIONAL CONVERGENCE

DEF'N: $\sum_{n=1}^{\infty} a_n$ CONV **ABSOLUTELY** IF $\sum_{n=1}^{\infty} |a_n|$ CONV

(IE. IF MAKING ALL TERMS POSITIVE GIVES A SERIES THAT CONV, THEN THE ORIGINAL SERIES (WITH A MIX OF SIGNS) IS SAID TO CONV ABSOLUTELY)

DOES $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONV ABSOLUTELY ?

$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n}$ DIV (~~DIV TEST~~ p-SERIES)

$p = 1 \leq 1$

SO $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ DOES NOT CONV ABSOLUTELY

BUT $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONV, SO WE SAY $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONV CONDITIONALLY

DEF'N: IF $\sum_{n=1}^{\infty} a_n$ CONV BUT DOES NOT CONV ABSOLUTELY, $\sum_{n=1}^{\infty} a_n$ CONV CONDITIONALLY